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II. Solution by W. W. BEMAN, A. M., Professor of Mathematics, State University, Ann Arbor, Mich.

$$\left(\tan^{-1}x - \frac{x}{1+x^2}\right) \frac{d^2y}{dx^2} = \frac{2x}{(1+x^2)^2} \left(x \frac{dy}{dx} - y\right).$$

Writing the equation in the form

$$\frac{d}{dx} \left(\frac{\tan^{-1}x}{x} \right) \cdot \frac{d^2y}{dx^2} = \frac{d}{dx} \left(\frac{1}{1+x^2} \right) \cdot \frac{d}{dx} \left(\frac{y}{x} \right)$$

it is obvious that $y = \tan^{-1}x$ and $y = x$ are independent particular integrals. Hence the complete primitive is $y = c_1 \tan^{-1}x + c_2 x$.

Solutions of this problem were also received from L. C. WALKER, and G. B. M. ZERR.

126. Proposed by JOHN M. COLAW, A. M., Monterey, Va.

Find the volume contained between the conical surface whose equation is $z = a - \sqrt{x^2 + y^2}$, and the planes whose equations are $x = z$ and $x = 0$ by the formula $\iiint dx dy dz$. [Todhunter's *Integral Calculus*.]

Solution by W. J. GREENSTREET, M. A., Editor of The Mathematical Gazette, Stroud, Gloucestershire, England.

The cone clearly extends from vertex $(0, 0, a)$ towards $z = 0$. Hence in

$$\int_{x_1}^{x_2} \int_{y_1}^{y_2} \int_{z_1}^{z_2} dx dy dz$$

we have $z_1 = x$; $z_2 = a - \sqrt{x^2 + y^2}$; $x = a - \sqrt{x^2 + y^2}$.

$\therefore y^2 = a^2 - 2ax$, $x_1 = 0$, and for x_2 we have $y_1 = y_2$. $\therefore x = \frac{1}{2}a$.

$$\begin{aligned} \therefore \text{Volume} &= \int_0^{\frac{1}{2}a} \int_{y_1}^{y_2} [a - x - \sqrt{x^2 + y^2}] dx dy \\ &= \int_0^{\frac{1}{2}a} \left[(a-x)(y_2 - y_1) - \frac{y}{2} \sqrt{x^2 + y^2} - \frac{x^2}{2} \log y + \sqrt{x^2 + y^2} \right]_{y_1}^{y_2} dx \\ &= \int_0^{\frac{1}{2}a} \left[2(a-x) \sqrt{a^2 - 2ax} - \sqrt{a^2 - 2ax}(a-x) - \frac{x^2}{2} \log \frac{a-x + \sqrt{a^2 - 2ax}}{a-x - \sqrt{a^2 - 2ax}} \right] dx. \end{aligned}$$

Put $2x = a \sin^2 \phi$.

$$\begin{aligned} \therefore V &= \int_0^{\frac{1}{2}\pi} a \sin \phi \cos \phi d\phi \left[(1 + \cos \phi) \frac{a^2}{2} \cos \phi - \frac{a^2}{8} \sin^4 \phi \log \left(\frac{1 + \cos \phi}{1 - \cos \phi} \right)^2 \right] \\ &= \frac{a^3}{2} \int_0^{\frac{1}{2}\pi} \left[\sin \phi \cos^2 \phi + \sin \phi \cos^4 \phi - \sin^5 \phi \cos \phi \log \frac{1 + \cos \phi}{\sin \phi} \right] d\phi \end{aligned}$$

$$= \frac{a^3}{2} \left(\frac{1}{3} + \frac{1}{5} \right) - \left[\frac{a^3 \sin^6 \phi}{12} \log \frac{1 + \cos \phi}{\sin \phi} \right]_0^{\frac{1}{2}\pi} - \frac{a^3}{12} \int_0^{\frac{1}{2}\pi} \sin^6 \phi \left(\frac{\cos \phi}{\sin \phi} + \frac{\sin \phi}{1 + \cos \phi} \right) dx$$

$$= \frac{4}{15} a^3 - \frac{1}{12} a^3 \int_0^{\frac{1}{2}\pi} [\cos \phi \sin^5 \phi + \sin^5 \phi (1 - \cos \phi)] d\phi = \frac{4}{15} a^3 - \frac{a^3}{12} \cdot \frac{4.2}{5.3} = \frac{2}{3} a^3.$$

Similar solutions were received from *GEORGE LILLEY*, *LON C. WALKER*, *G. B. M. ZERR*, and *J. SCHEFFER*.

MECHANICS.

127. Proposed by *F. P. MATZ*, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College, Defiance, Ohio.

Develop the Fourier Series to represent the temperature of a circular wire of uniform cross-section, in which the temperatures of the four quadrants are in order $t, 2t, 3t, 4t$.

Solution by *G. B. M. ZERR*, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

From Fourier's *Analytical Theory of Heat*, page 217, we get for the complete solution of the linear and varied movement of heat in a ring after a time T the following:

$$v = e^{-hT} \left[\frac{1}{2\pi} \int_0^{2\pi} f(x) dx + \frac{1}{\pi} \sum_{n=1}^{n=\infty} e^{-n^2 k T} \sin nx \int_0^{\frac{1}{2}\pi} \sin nx f(x) dx \right. \\ \left. + \frac{1}{\pi} \sum_{n=1}^{n=\infty} e^{-n^2 k T} \cos nx \int_0^{\frac{1}{2}\pi} \cos nx f(x) dx \right],$$

$$\text{where } f(x) = \frac{1}{2\pi} \int_0^{2\pi} f(x) dx + \frac{1}{\pi} \sum_{n=1}^{n=\infty} \sin nx \int_0^{2\pi} \sin nx f(x) dx \\ + \frac{1}{\pi} \sum_{n=1}^{n=\infty} \cos nx \int_0^{2\pi} \cos nx f(x) dx.$$

$$\int_0^{2\pi} f(x) dx = t \int_0^{\frac{1}{2}\pi} dx + 2t \int_{\frac{1}{2}\pi}^{\pi} dx + 3t \int_{\pi}^{\frac{3}{2}\pi} dx + 4t \int_{\frac{3}{2}\pi}^{2\pi} dx = 5\pi t.$$

$$\int_0^{2\pi} \sin nx f(x) dx = t \int_0^{\frac{1}{2}\pi} \sin nx dx + 2t \int_{\frac{1}{2}\pi}^{\pi} \sin nx dx + 3t \int_{\pi}^{\frac{3}{2}\pi} \sin nx dx + 4t \int_{\frac{3}{2}\pi}^{2\pi} \sin nx dx$$

$$= \frac{t}{n} (\cos \frac{1}{2}\pi n + \cos \pi n + \cos \frac{3}{2}\pi n) - \frac{3t}{n} = -\frac{4t}{n}, \text{ except when } n=4m.$$

$$\int_0^{2\pi} \cos nx f(x) dx = t \int_0^{\frac{1}{2}\pi} \cos nx dx + 2t \int_{\frac{1}{2}\pi}^{\pi} \cos nx dx + 3t \int_{\pi}^{\frac{3}{2}\pi} \cos nx dx + 4t \int_{\frac{3}{2}\pi}^{2\pi} \cos nx dx$$